



Reconnaissance des Formes et Perception
Intelligence Artificielle

Atelier - Perception pour le véhicule intelligent (P. Vasseur)

Visual Servoing and Exploration for Mobile Robots

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Motivation

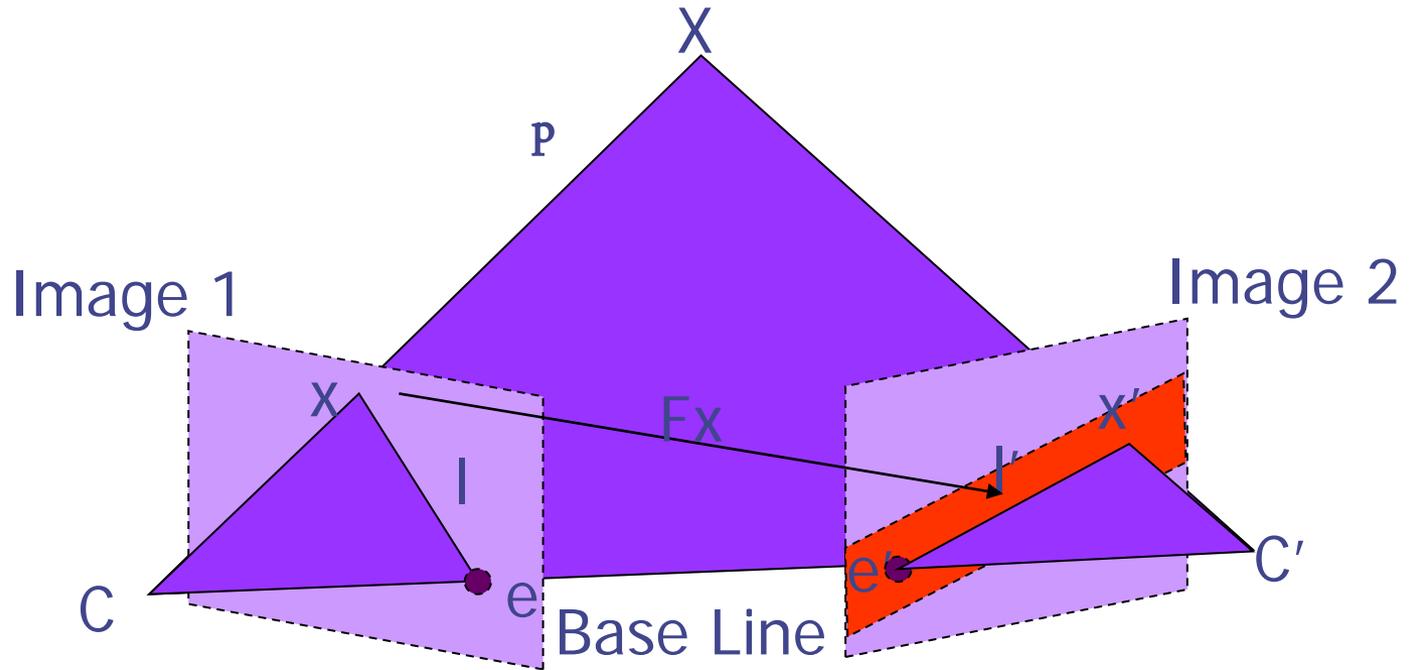


Index

- ◆ Long term navigation using **epipolar geometry**
- ◆ Multi-robot control using **homography**
 - One / Several cameras

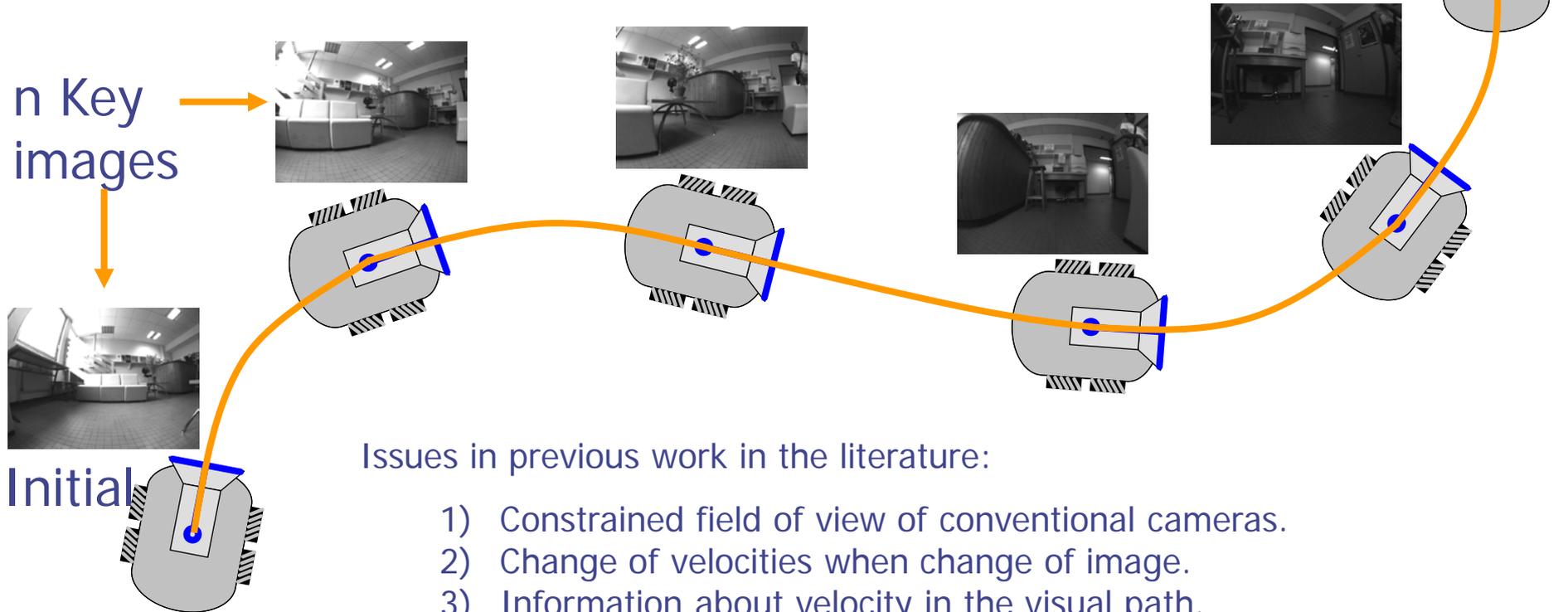
“Visual control” using
“Geometric constraints”

Long term navigation



Long term navigation

- Task: reach a desired position associated with a target image, which belongs to a visual memory acquired in a teaching phase.
- A visual path of n key images is extracted from the visual memory, which must be followed autonomously in order to reach the target.

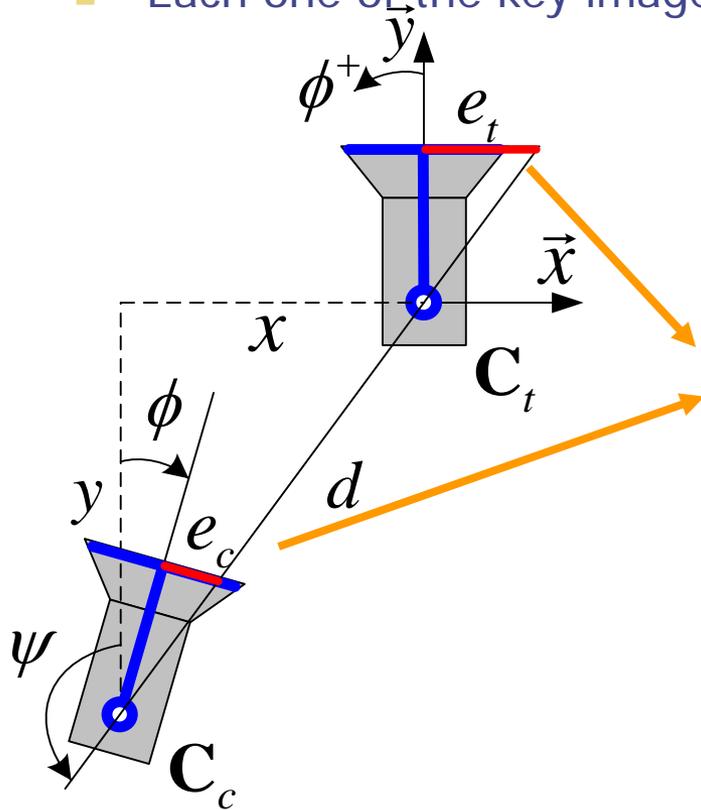


Issues in previous work in the literature:

- 1) Constrained field of view of conventional cameras.
- 2) Change of velocities when change of image.
- 3) Information about velocity in the visual path.

Long term navigation

- The omnidirectional cameras can be virtually represented as conventional cameras when working with points on the sphere.
- Each one of the key images is used as target image accordingly.



Target location

$$\Rightarrow \mathbf{C}_t = (0,0,0)$$

Current location

$$\Rightarrow \mathbf{C}_c = (x, y, \phi)$$

Epipoles

$$\left\{ \begin{array}{l} e_c = \alpha_x \frac{x \cos \phi + y \sin \phi}{y \cos \phi - x \sin \phi}, \\ e_t = \alpha_x \frac{x}{y}. \end{array} \right.$$

- Interaction with the robot velocities:

$$\dot{e}_c = -\frac{\alpha_x \sin(\phi - \psi)}{d \cos^2(\phi - \psi)} v + \frac{\alpha_x}{\cos^2(\phi - \psi)} \omega,$$

$$\dot{e}_t = -\frac{\alpha_x \sin(\phi - \psi)}{d \cos^2(\psi)} v$$

Long term navigation

- Let us define a tracking error to drive the epipole smoothly to zero for every segment between key images

$$\zeta_{ce} = e_c - e_c^d(t) = 0.$$

where $e_c^d(t) = \frac{e_c(0)}{2} \left(1 + \cos\left(\frac{\pi}{\tau} t\right) \right), 0 \leq t \leq \tau$ with $\tau = \frac{d_{\min}}{v}$.

$$e_c^d(t) = 0, \quad t > \tau$$

- Control goal – Stabilization of the error system:

$$\dot{\zeta}_{ce} = -\frac{\alpha_x \sin(\phi - \psi)}{d \cos^2(\phi - \psi)} v + \frac{\alpha_x}{\cos^2(\phi - \psi)} \omega_{rt}^{ce} - \dot{e}_c^d.$$

- Considering that the translational velocity is known, the following rotational velocity, referred as **reference tracking (RT) control**, stabilizes the error system

$$\omega_{rt}^{ce} = \frac{\sin(\phi - \psi)}{d} v + \frac{\cos^2(\phi - \psi)}{\alpha_x} (\dot{e}_c^d - k_c \zeta_{ce}).$$

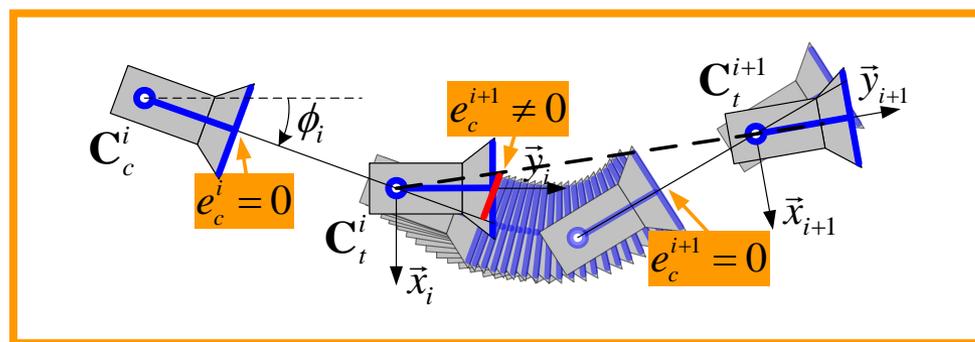
with $k_c > 0$.

Long term navigation

- The current epipole gives information of the translation direction and it is directly related to the required robot rotation to be aligned with the target.
- Use of the x -coordinate of the current epipole as feedback information to control the robot heading and so, to correct the lateral deviation.

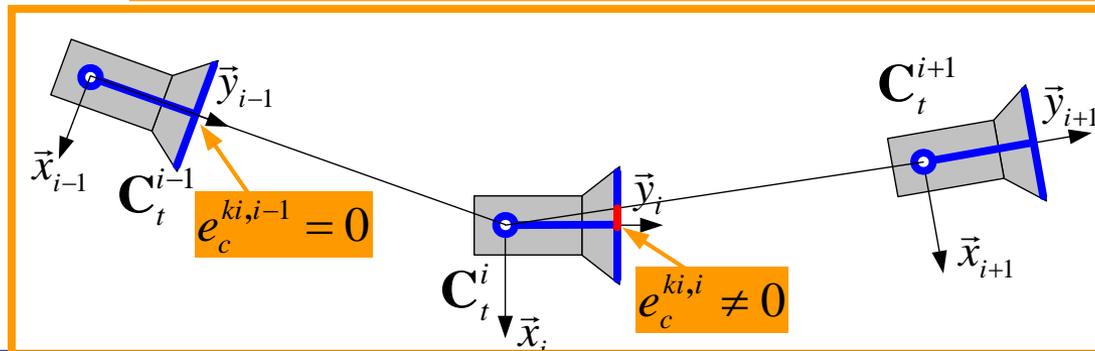
Non-null translational velocity $v \neq 0$ Rotational velocity $\omega^{ce} = k_t \omega_{rt}^{ce} + \bar{\omega}^{ce}$

First component of the rotational velocity



$$\omega \Rightarrow f(e_c)$$

Second component of the rotational velocity



$$\omega \Rightarrow f(e_c^{ki})$$

Long term navigation

- A varying translational velocity according to the shape of the path can be computed depending on the epipoles between key images.

$$v^{ce} = v_{\max} + v_{\min} + \frac{v_{\max} - v_{\min}}{2} \tanh\left(1 - \frac{|e_c^{ki} / d_{\min}|}{\sigma}\right). \quad \tau = \frac{d_{\min}}{v}$$

- We propose the following nominal rotational velocity, which is computed from the epipoles between key images:

$$\bar{\omega}^{ce} = \frac{k_m v^{ce}}{d_{\min}} e_c^{ki}.$$

- So that, the complete rotational velocity (RT+ control) is given as:

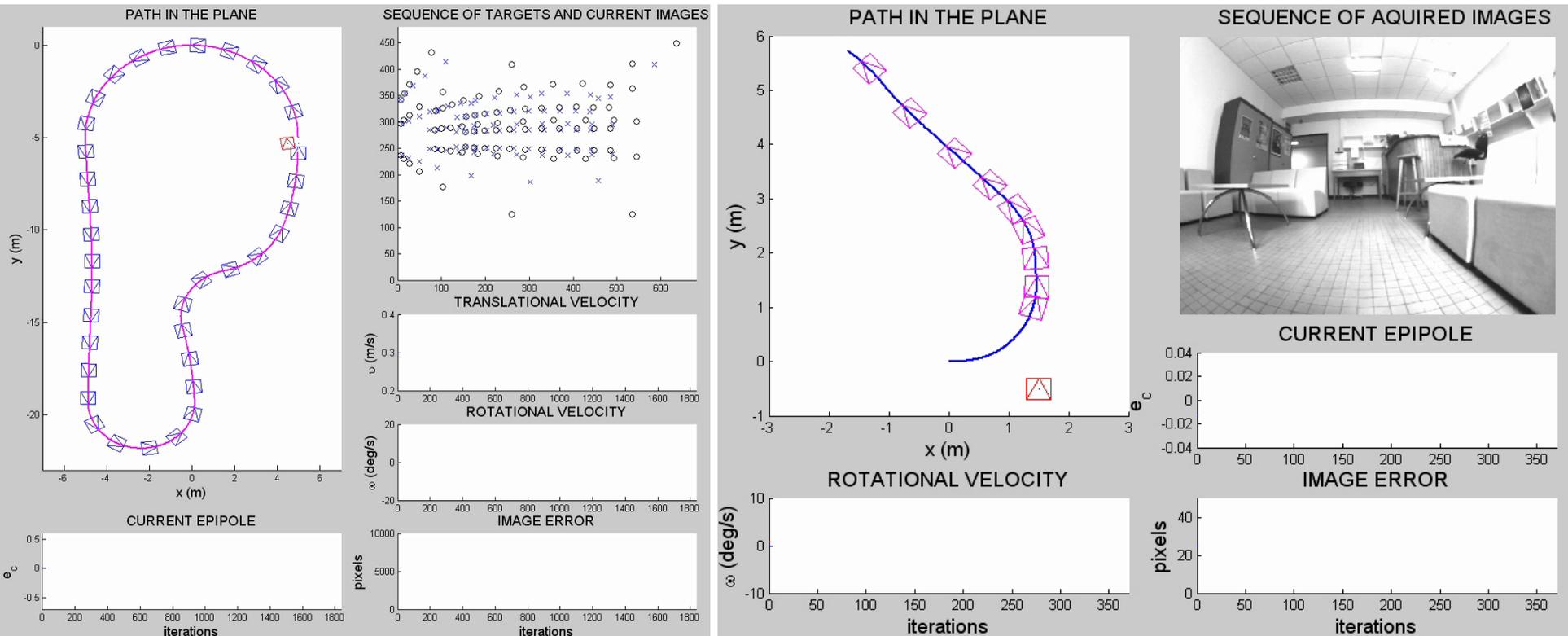
$$\omega^{ce} = k_t \omega_{rt}^{ce} + \bar{\omega}^{ce}.$$

Switching and stop condition

- The switching condition to the next key image or to stop the task is given when the image error starts to increase, which is defined as follows:

$$\varepsilon = \frac{1}{r} \sum_{j=1}^r \|\mathbf{p}_j - \mathbf{p}_{i,j}\|.$$

Long term navigation



Wheeled mobile robots navigation from a visual memory using wide field of view cameras,
 H.M. Becerra, J. Courbon, Y. Mezouar, C. Sagues, IEEE/RSJ Int. Conference on Intelligent Robots and Systems (IROS'10), pages 5693-5699, Taipei, Taiwan, October 18-22, 2010

Visual navigation of wheeled mobile robots using direct feedback of a geometric constraint
 H.M. Becerra, C. Sagues, Y. Mezouar, J.-B. Hayet, Autonomous Robots, (ISSN 0929-5593),
 d.o.i. 10.1007/s10514-014-9382-3, Vol. 37(2):137-156 , 2014.

Index

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“Visual control” using
“Geometric constraints”

Multi robot control with flying camera (H)

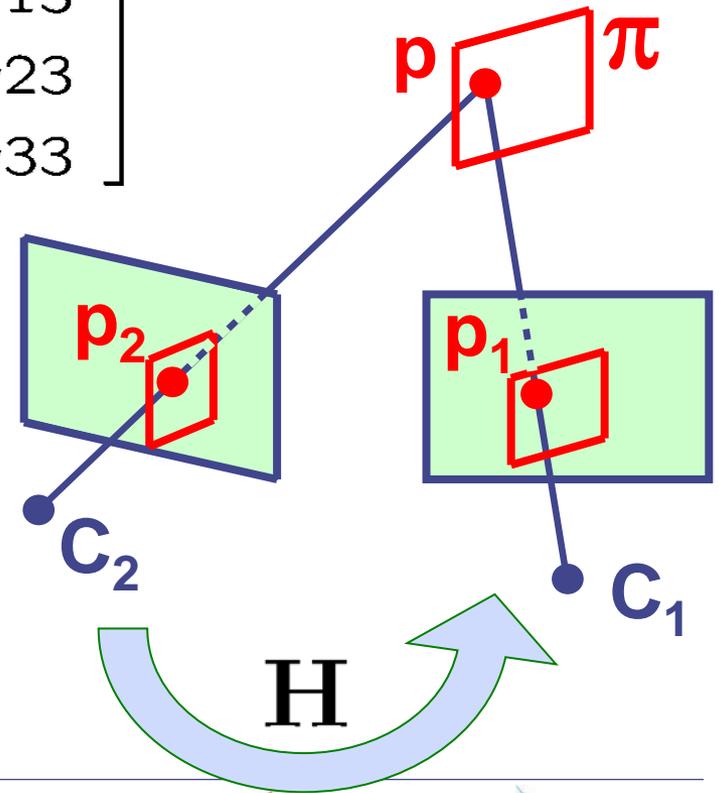
- ◆ Homography: Projective transformation relating the projections in two perspective images of points belonging to a scene plane

$$\mathbf{p}_2 = \mathbf{H} \mathbf{p}_1 \quad \mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

- Can be computed linearly (least squares) from four point matches
- Gives the motion (rotation \mathbf{R} and translation \mathbf{T}) between the cameras:

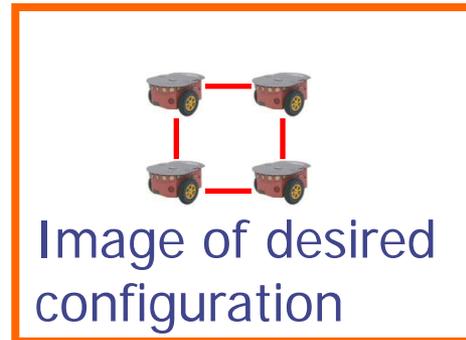
$$\mathbf{H} = \mathbf{R} + \mathbf{T} \mathbf{n}^T / d$$

Plane: (\mathbf{n}, d)



Multi robot control with flying camera (H)

- ◆ What? Visual control of mobile robots
 - Desired configuration defined by an image
 - Task: Navigate to the desired configuration



Initial configuration

Desired configuration

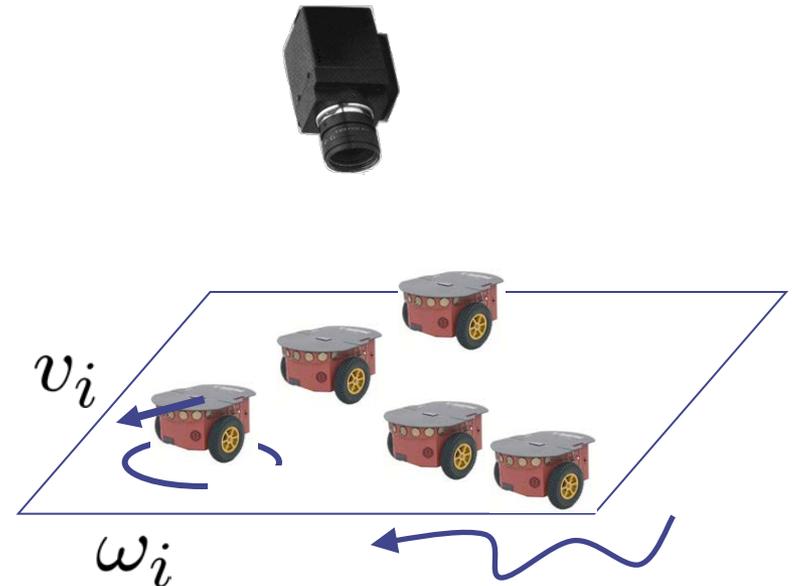
Multi robot control with flying camera (H)

- ◆ What? Visual control of mobile robots
- ◆ Who? Set of nonholonomic vehicles
 - Nonholonomic kinematics
 - ◆ Cartesian coordinates

$$\begin{aligned}\dot{x} &= -v \sin \phi \\ \dot{y} &= v \cos \phi \\ \dot{\phi} &= \omega\end{aligned}$$

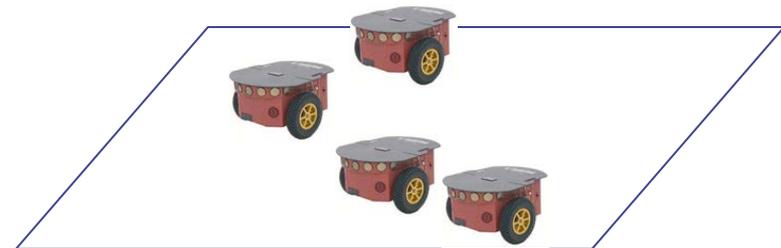
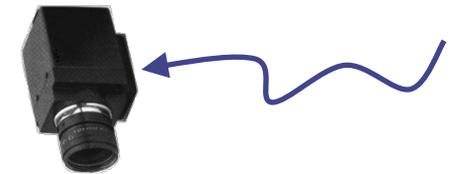
- ◆ Polar coordinates

$$\begin{aligned}\dot{\rho} &= v \cos \alpha \\ \dot{\alpha} &= \omega - \frac{v}{\rho} \sin \alpha \\ \dot{\phi} &= \omega\end{aligned}$$



Multi robot control with flying camera (H)

- ◆ What? Visual control of mobile robots
- ◆ Who? Set of nonholonomic vehicles
- ◆ How? Flying camera
 - ◆ Flying camera looking downward
 - ◆ Camera motion unknown
 - ◆ Intrinsic camera parameters known
 - ◆ Homography: Only visual information

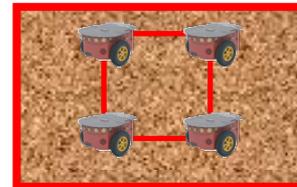


Multi robot control with flying camera (H)

- ◆ What? Visual control of mobile robots
- ◆ Who? Set of nonholonomic vehicles
- ◆ How? Flying camera
- ◆ Where? Motion occurs in a planar floor
 - This gives additional constraints on the homography
 - Only the set of robots may remain common in the scene



Image of desired configuration:



Desired configuration



Actual configuration



Multi robot control with flying camera (H)

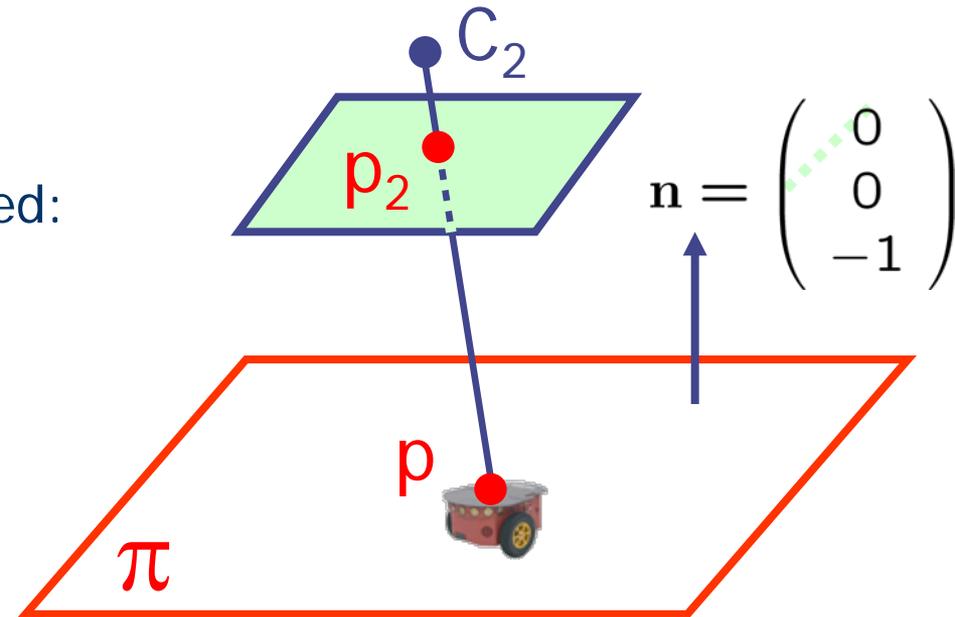
◆ The homography in our framework:

- Multi-robot motion in a planar floor
- Points = Robots => Homography
- Camera flies parallel to the floor

◆ Then, the homography is constrained:

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} \cos \phi & \sin \phi & -t_x/d \\ -\sin \phi & \cos \phi & -t_y/d \\ 0 & 0 & 1 \end{bmatrix}$$



◆ This homography can be computed from a minimal set of two points/robots

Multi robot control with flying camera (H)

H_{rigid}

◆ If the robots are in the desired configuration:

- The homography is conjugate to a planar Euclidean transformation
- The homography is not the identity matrix

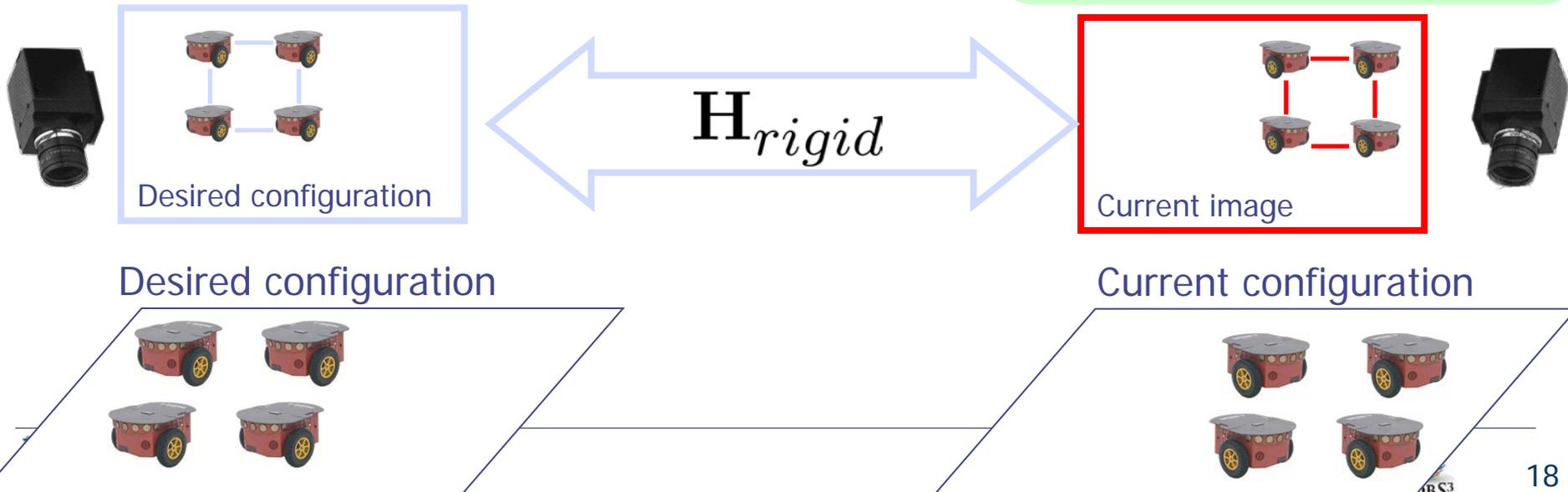
$$H_{rigid} = \begin{bmatrix} \cos \phi & \sin \phi & h_{13} \\ -\sin \phi & \cos \phi & h_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Extracting Motion parameters

$$\mathbf{n} = (0, 0, -1)^T$$

$$\mathbf{x} = (x, y, 0)^T$$

Which is coherent with a rigid motion. So, the robots are in the desired formation



Multi robot control with flying camera (H)

$H_{nonrigid}$

◆ If the robots are **NOT** in the desired configuration:

- The homography is a similarity transformation with isotropic scaling s
- The H computation with the 2-point method

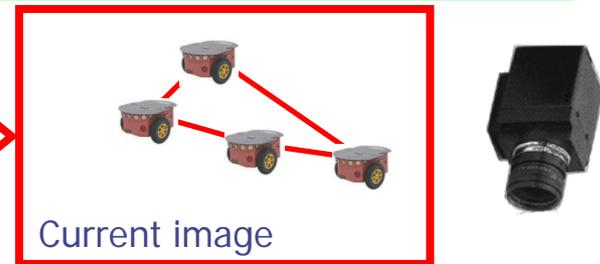
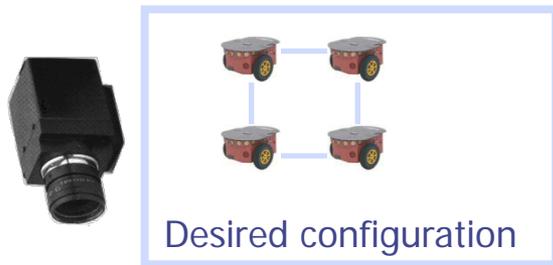
$$H_{nonrigid} = \begin{bmatrix} s \cos \phi & s \sin \phi & h_{13} \\ -s \sin \phi & s \cos \phi & h_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Extracting Motion parameters

$$\mathbf{n} = (0, 0, -1)^T$$

$$\mathbf{x} = (x, y, (s-1)d^2)^T$$

Which is **NOT** coherent with a rigid motion. So, the robots are not in formation



Desired configuration



Current configuration



Multi robot control with flying camera (H)



- ◆ We have
 - Robots not in formation
 - Nonrigid homography
 - Each pair of robots induces a different Homography, valid but not coherent

$$\mathbf{H}_{nonrigid} = \begin{bmatrix} s \cos \phi & s \sin \phi & h_{13} \\ -s \sin \phi & s \cos \phi & h_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{p}' = \mathbf{H}_{nonrigid} \mathbf{p}$$

- ◆ We want
 - Robots in formation
 - Rigid homography
 - Every pair of robots induce the same Homography

$$\mathbf{H}_{rigid} = \begin{bmatrix} \cos \phi & \sin \phi & h_{13} \\ -\sin \phi & \cos \phi & h_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

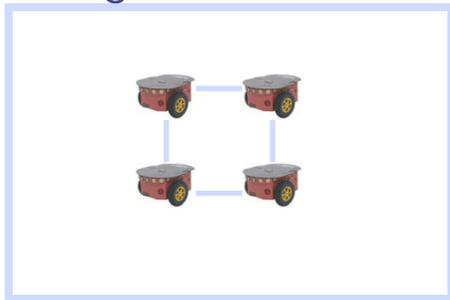
- ◆ We define a desired homography
 - Like the nonrigid homography but being induced by keeping the motion constraints
 - The task is to drive the robots to the desired homography
 - The desired homography is not constant and depends on the robots and camera motion

$$\mathbf{H}^d = \mathbf{H}_{nonrigid} \begin{bmatrix} 1/s & 0 & 0 \\ 0 & 1/s & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

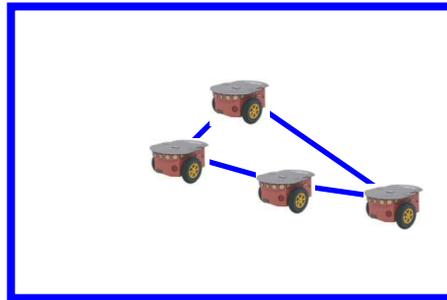
$$\mathbf{p}^d = (\mathbf{H}^d)^{-1} \mathbf{p}'$$

Multi robot control with flying camera (H)

Image of desired configuration



Current image



Flying camera



Desired H^d homography

Control law

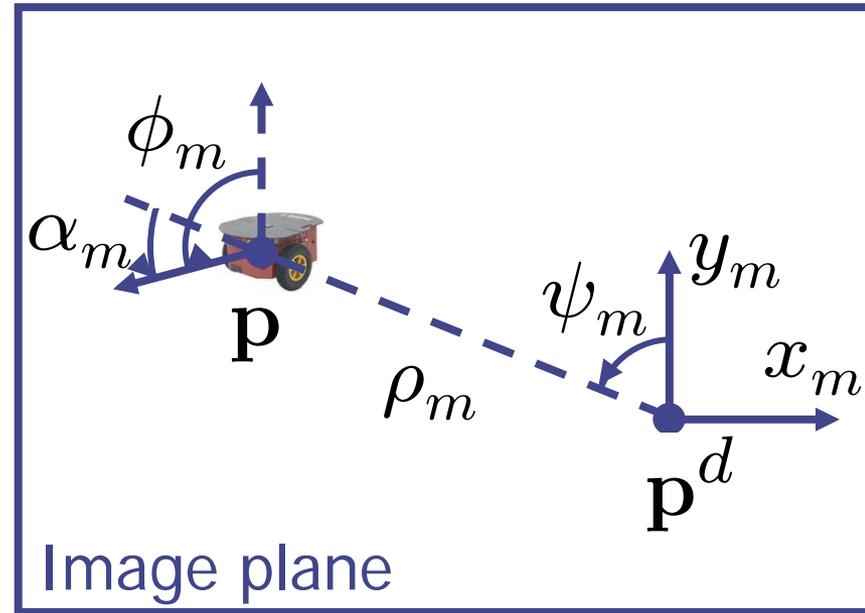
Set of robots:

$$\begin{pmatrix} v_i \\ \omega_i \end{pmatrix}$$



Multi robot control with flying camera (H)

- ◆ Image-based control law
- ◆ Control error:
 - Current state of the robots on the image vs desired states given by the desired homography
- ◆ Switched control consisting of three sequential steps:



$$\text{Step 1 } \begin{cases} v = 0 \\ \omega = \dot{\psi}_c - k_\omega (\alpha_m - \pi) \end{cases}$$

$$\text{Step 2 } \begin{cases} v = \dot{\rho}_d - k_v \rho_m \\ \omega = \dot{\psi}_c - k_\omega (\alpha_m - \pi) \end{cases}$$

$$\text{Step 3 } \begin{cases} v = 0 \\ \omega = -k_\omega ((\phi_m - \psi_{Fm}) - (\phi_m^0 - \psi_{Fm}^0)) \end{cases}$$

Stability for the control can be demonstrated

$$\rho_m = \sqrt{(p_x - p_x^d)^2 + (p_y - p_y^d)^2}$$

$$\psi_m = \text{atan2}(-(p_x - p_x^d), (p_y - p_y^d))$$

$$\psi_{Fm} = \text{atan2}(-(p_x^i - p_x^j), (p_y^i - p_y^j))$$

$$\mathbf{x}^d(t) = (x^d, y^d, \phi^d)^T$$

$$\dot{\rho}_d = \partial \rho_c / \partial \mathbf{x}^d$$

$$\phi_m = \text{atan2}(-\Delta p_x, \Delta p_y)$$

$$\alpha_m = \phi_m - \psi_m$$

Multi robot control with flying camera (H)

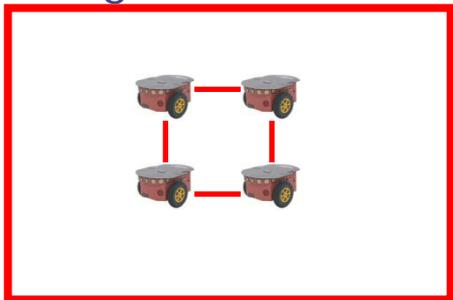
- ◆ Steps 1-2 orientate and drive the robots toward their target locations. In practice, they are carried out simultaneously:

$$\text{Step 1 and 2} \begin{cases} v = \dot{\rho}_d - k_v \rho_m \\ \omega = \dot{\psi}_c - k_\omega (\alpha_m - \pi) \end{cases}$$

- ◆ Step 3 rotates the robots until they are in the required relative orientation within the formation

$$\text{Step 3} \begin{cases} v = 0 \\ \omega = -k_\omega ((\phi_m - \psi_{Fm}) - (\phi_m^0 - \psi_{Fm}^0)) \end{cases}$$

Image of desired configuration



Steps
1-2



Step
3



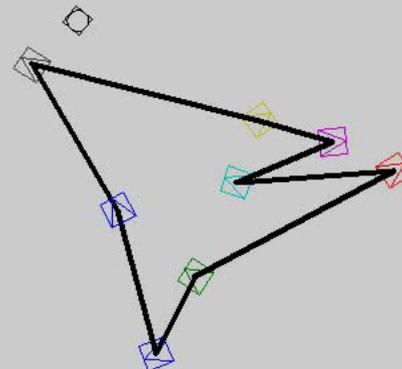
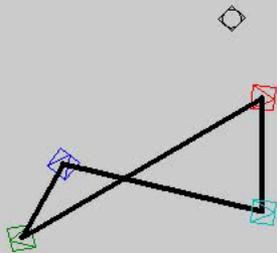
Multi robot control with flying camera (H)

Top view

Linear velocity: v

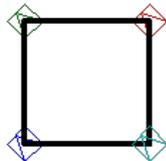
Top view

Homography entries

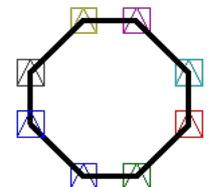


Angular velocity: ω

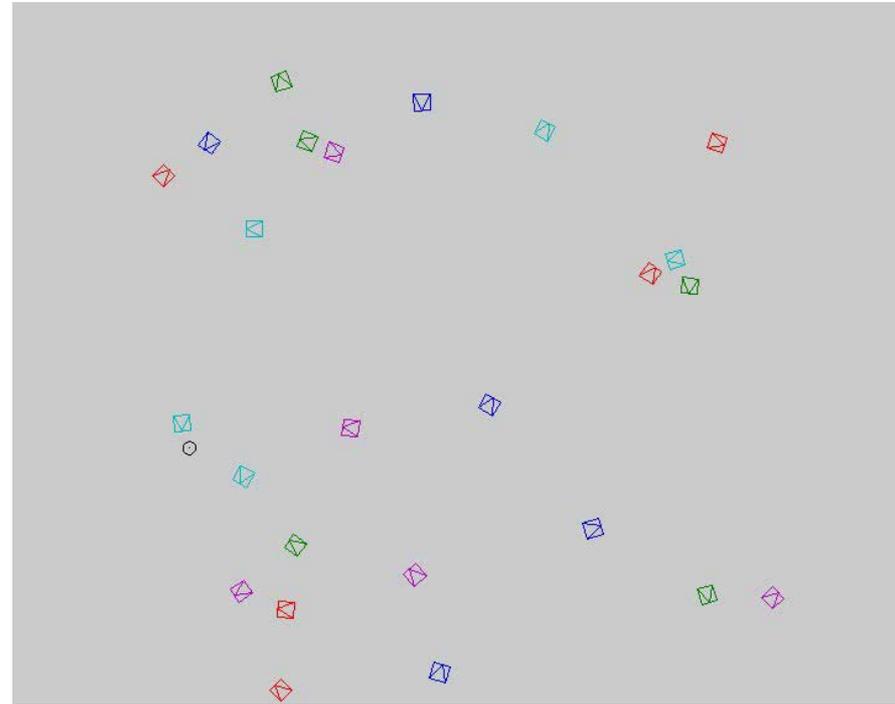
Desired configuration:



Desired configuration:



Multi robot control with flying camera (H)



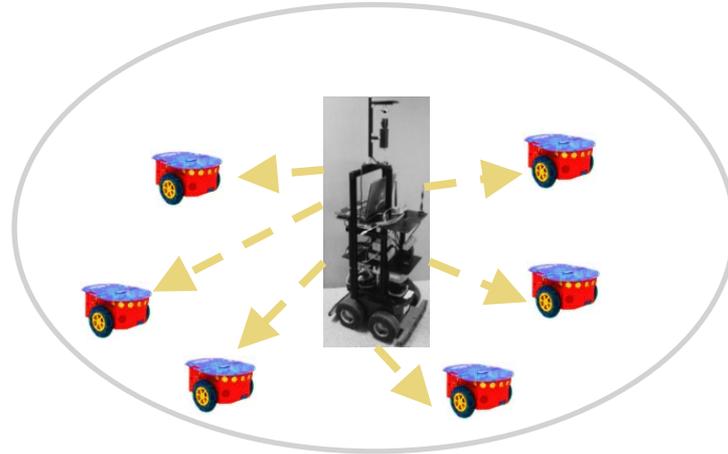
Homography-Based Multi-Robot Control with a Flying Camera, G. López-Nicolás, Y. Mezouar, C. Sagues, IEEE Int. Conference on Robotics and Automation (ICRA'11), pages 4492-4497, Shanghai-China, May 9-13, 2011.

Visual Control for Multi-Robot Organized Rendezvous, G. Lopez-Nicolas, M. Aranda, Y. Mezouar and C. Sagues, IEEE Transactions on Systems Man and Cybernetics: Part B. Vol.: 42(4):1155-1168, d.o.i. 10.1109/TSMCB.2012.2187639, 2012.

Multi robot control with multiple cameras (H)

Better performance:

- Minimization of sum squared distances current-desired position

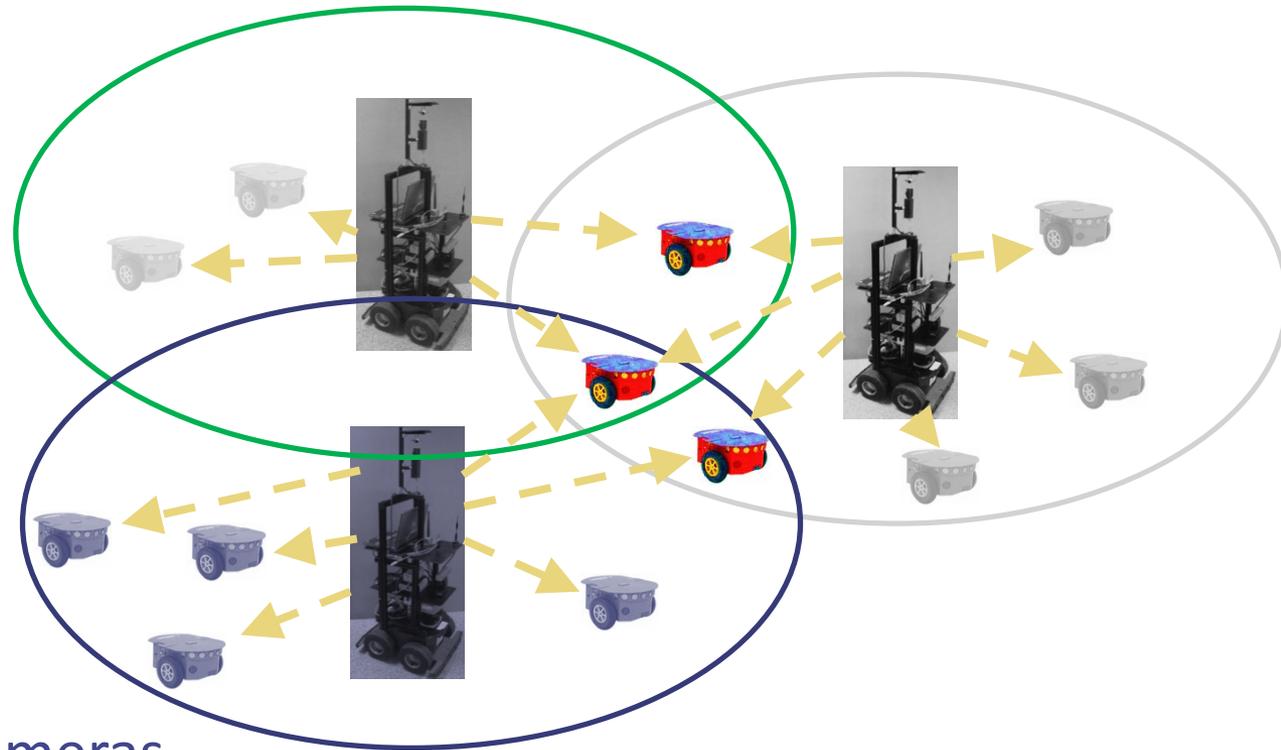


Several issues with the single-camera centralized method:

- ◆ Robustness (single point of failure)
- ◆ Sensing range (limited field of view)
- ◆ Scalability w.r.t. number of robots (processing and communications requirements)

Multiple cameras /Multiple robots

Multi robot control with multiple cameras (H)



- Omnidirectional cameras
- Each camera is one controller
- Each camera controls a subset of the team
- Distributed (partially)
- Communication camera - Controlled robots
- No communication between cameras

Multi robot control with multiple cameras (H)

- We constrain the homography to have the form of a Euclidean 2D transformation, also called a rigid transformation:

$$\mathbf{H}_e = \begin{bmatrix} \cos \phi_e & \sin \phi_e & t_{xe} \\ -\sin \phi_e & \cos \phi_e & t_{ye} \\ 0 & 0 & 1 \end{bmatrix} \quad \left\{ \begin{array}{l} \mathbf{t} = (t_{xe}, t_{ye})^T \\ \phi_e \end{array} \right.$$

- Observe that:
 - ✓ The assumed planar camera motion can be explained by this homography as a translation and rotation
 - ✓ Any rigid motion w.r.t. the reference image is also accounted for by this homography
 - ✓ Then, the errors of this mapping of points must be due only to a nonrigid motion of the robots, i.e. the motion that separates the set from being in formation

Multi robot control with multiple cameras (H)

- Method outline:
 1. Compute a least-squares Euclidean homography using the reference and current image points
 2. Use it to map the reference points to a new set of *desired* points in the current image
 3. Make the robots move so that the points in the current image coincide with the *desired* points
- We are not interested in computing the camera motion or any rigid motion of the robot set, only in isolating the effect of **nonrigid motions** of the group
- Note the camera can perform any **arbitrary planar motion**

Multi robot control with multiple cameras (H)

- ◆ We will move the robots so that their current image positions \mathbf{p} will coincide with those (\mathbf{p}^d) given by the homography mapping computed from \mathbf{p} and the points of the desired configuration, \mathbf{p}'
- ◆ An objective of interest can be:

$$\mathbf{p}^d(\mathbf{H}^d, \mathbf{p}', \mathbf{p}) \ni S = \sum_{i=1}^n \|\mathbf{p}_i - \mathbf{p}_i^d\|^2 \text{ is minimum}$$



A necessary condition is that the centroids of \mathbf{p} and \mathbf{p}^d coincide:

$$\mathbf{p}'_c = \mathbf{p}' - \mathbf{c}_{\mathbf{p}'}, \quad \mathbf{p}_c = \mathbf{p} - \mathbf{c}_{\mathbf{p}}$$

Multi robot control with multiple cameras (H)

◆ Then, we can simplify the homography parameterization:

$$\mathbf{H}_r = \begin{bmatrix} h_{11}^r & h_{12}^r & 0 \\ h_{21}^r & h_{22}^r & 0 \\ 0 & 0 & h_{33}^r \end{bmatrix} \sim \begin{bmatrix} \cos \phi_r & \sin \phi_r & 0 \\ -\sin \phi_r & \cos \phi_r & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (6)$$

- Constraints: $h_{11}^r = h_{22}^r$, $h_{12}^r = -h_{21}^r$, $h_{11}^r{}^2 + h_{12}^r{}^2 = 1$
- Considering only the two linear constraints:

$$\mathbf{H}_l = \begin{bmatrix} h_{11}^l & h_{12}^l & 0 \\ -h_{12}^l & h_{11}^l & 0 \\ 0 & 0 & h_{33}^l \end{bmatrix} \sim \begin{bmatrix} s \cos \phi_l & s \sin \phi_l & 0 \\ -s \sin \phi_l & s \cos \phi_l & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (8)$$

Multi robot control with multiple cameras (H)

- We show that the solution which provides the minimum squared distances for the robots can be obtained from the linearly constrained H

Proposition 1: Let \mathbf{p}' and \mathbf{p} be two sets of n image points. Let \mathbf{H}_r be the least-squares homography computed from \mathbf{p}' and \mathbf{p} having the form (6), and let \mathbf{H}_1 be the least-squares homography computed from \mathbf{p}' and \mathbf{p} having the form (8). Then, $\mathbf{H}_r = \mathbf{H}_1 \cdot \text{diag}(1/s, 1/s, 0)$.

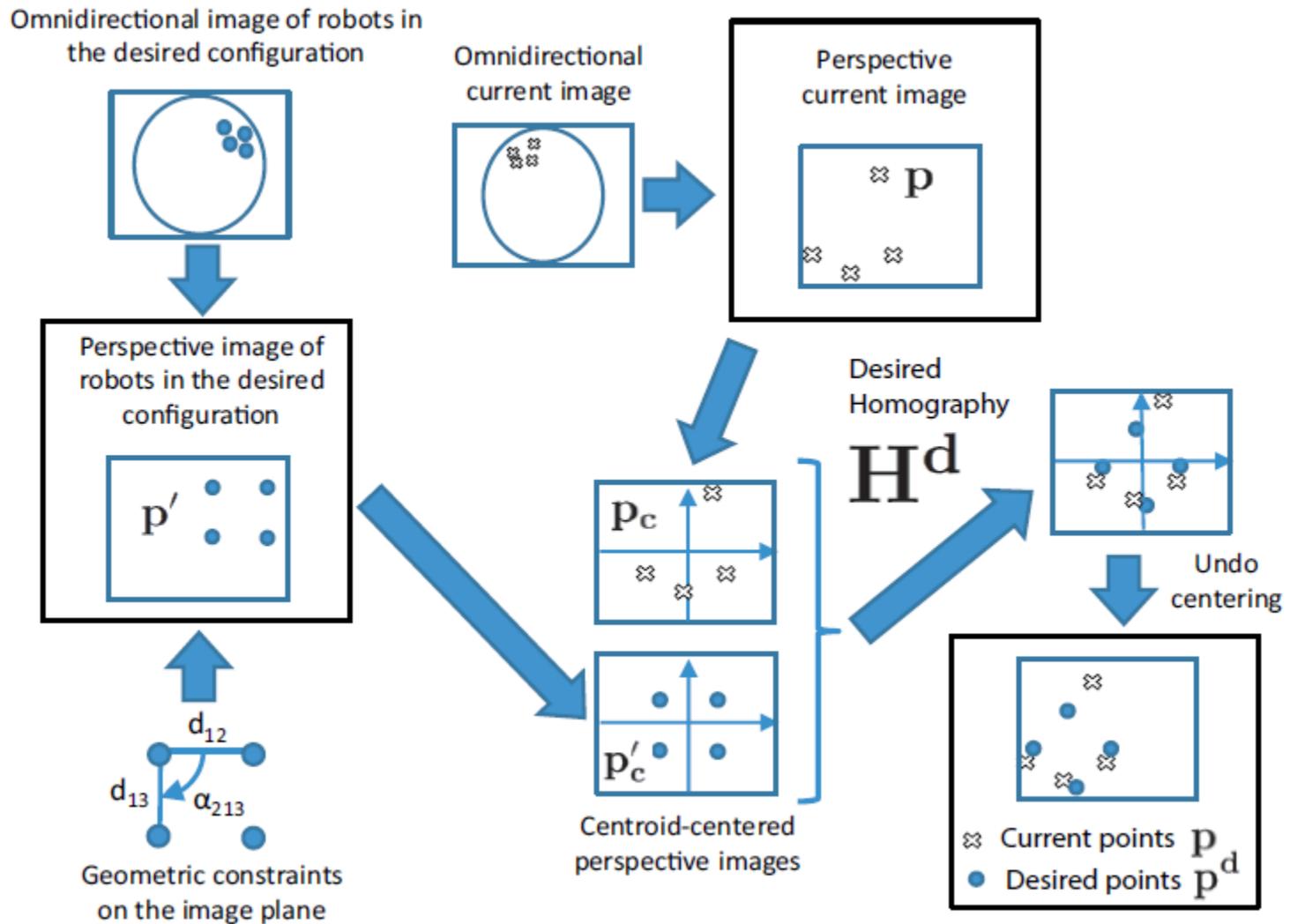
Multi robot control with multiple cameras (H)

- The solution is obtained efficiently: solving an overdetermined linear system with only one d.o.f.
- The desired image positions for the robots are then obtained from the computed homography ($H^d = Hr$):

$$Sr = \sum_{n=1}^{\infty} \left\| p_i - p_{ri}^d \right\|^2 = (p_{xi}'^2 + p_{yi}'^2) + (p_{xi}^2 + p_{yi}^2) - 2[\cos\phi_r(p_{xi}'p_{xi} - p_{yi}'p_{yi}) + \sin\phi_r(p_{yi}'p_{xi} - p_{xi}'p_{yi})]$$

$$p^d = H^d p'_c + c_p.$$

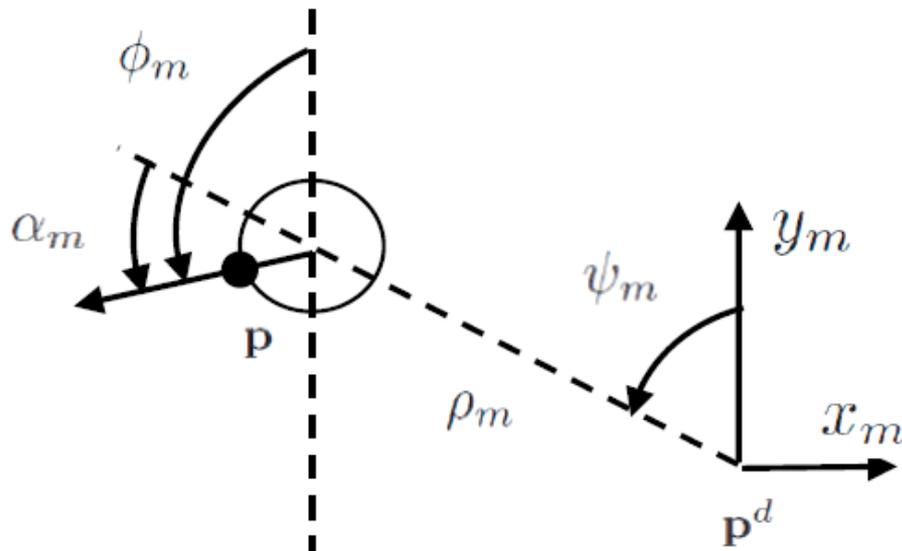
Multi robot control with multiple cameras (H)



Multi robot control with multiple cameras (H)

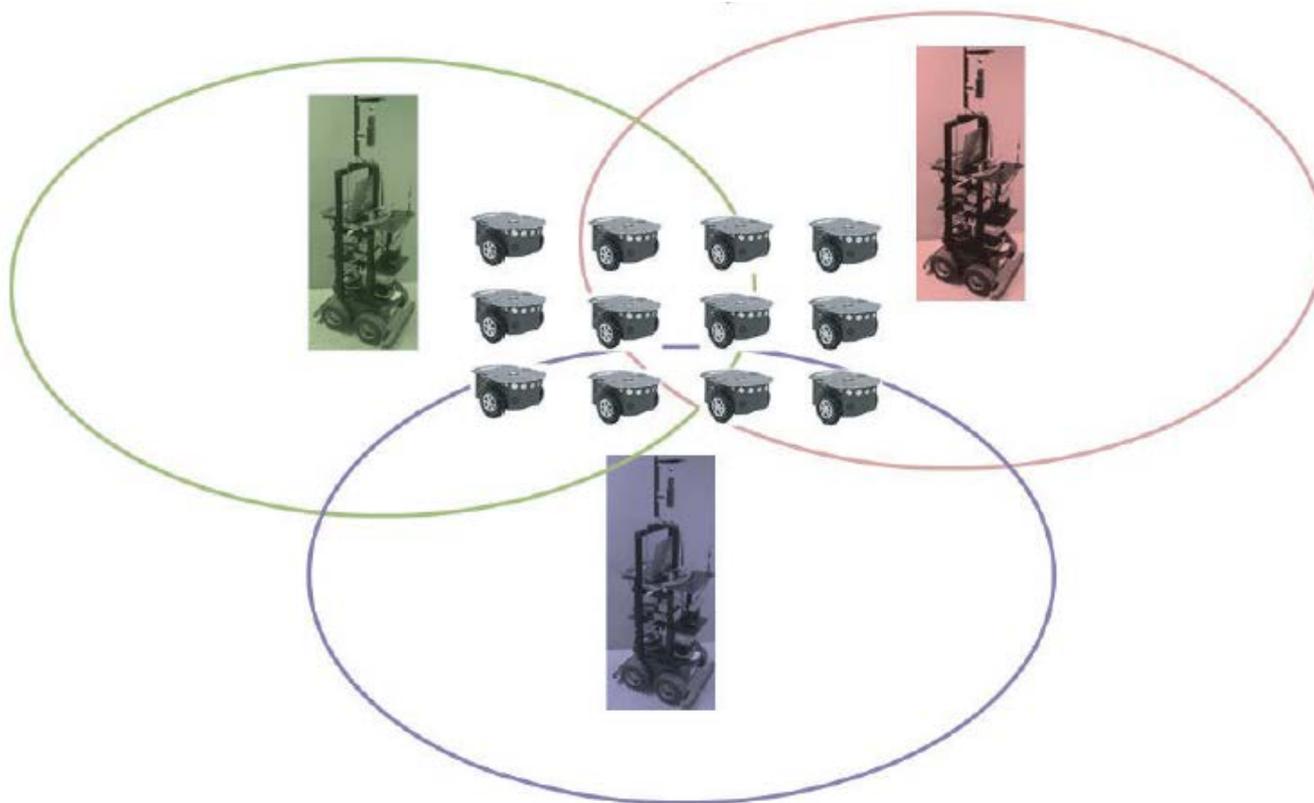
- An image-based control law with proven local exponential stability is defined to drive each of the unicycle robots to its desired position:

$$\begin{cases} v = -k_v \operatorname{sign}(\cos \alpha_m) \rho_m \\ \omega = k_\omega (\alpha_d - \alpha_m) \end{cases} \quad \alpha_d = \begin{cases} 0 & \text{if } |\alpha_m| \leq \frac{\pi}{2} \\ \pi & \text{if } |\alpha_m| > \frac{\pi}{2} \end{cases}$$



Multi robot control with multiple cameras (H)

- Overlaps between subsets are required.
- If every subset shares at least two robots with others, then convergence of the subsets implies convergence of the full set.



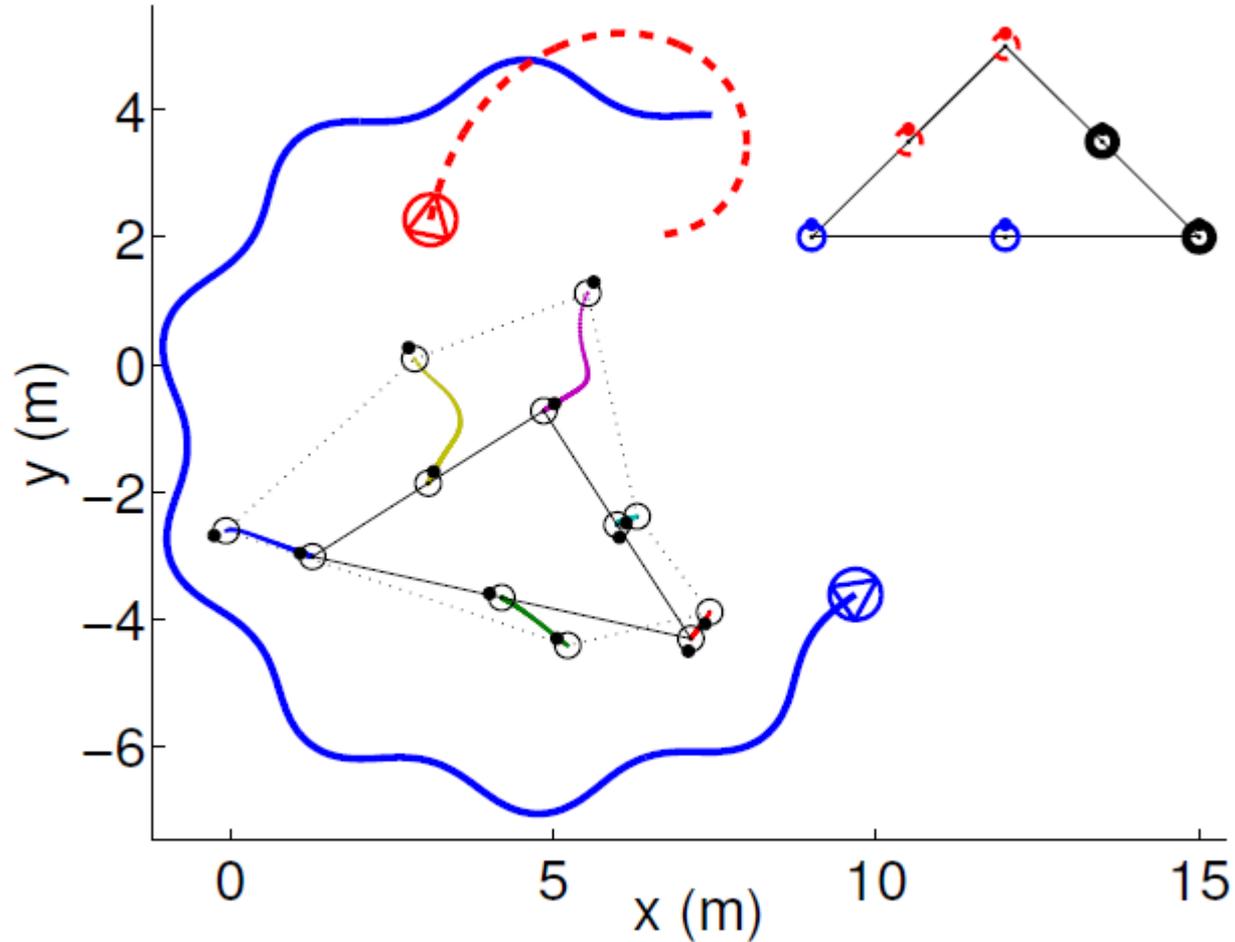
Multi robot control with multiple cameras (H)

Behavior of a robot that receives multiple motion commands?

Different strategies:

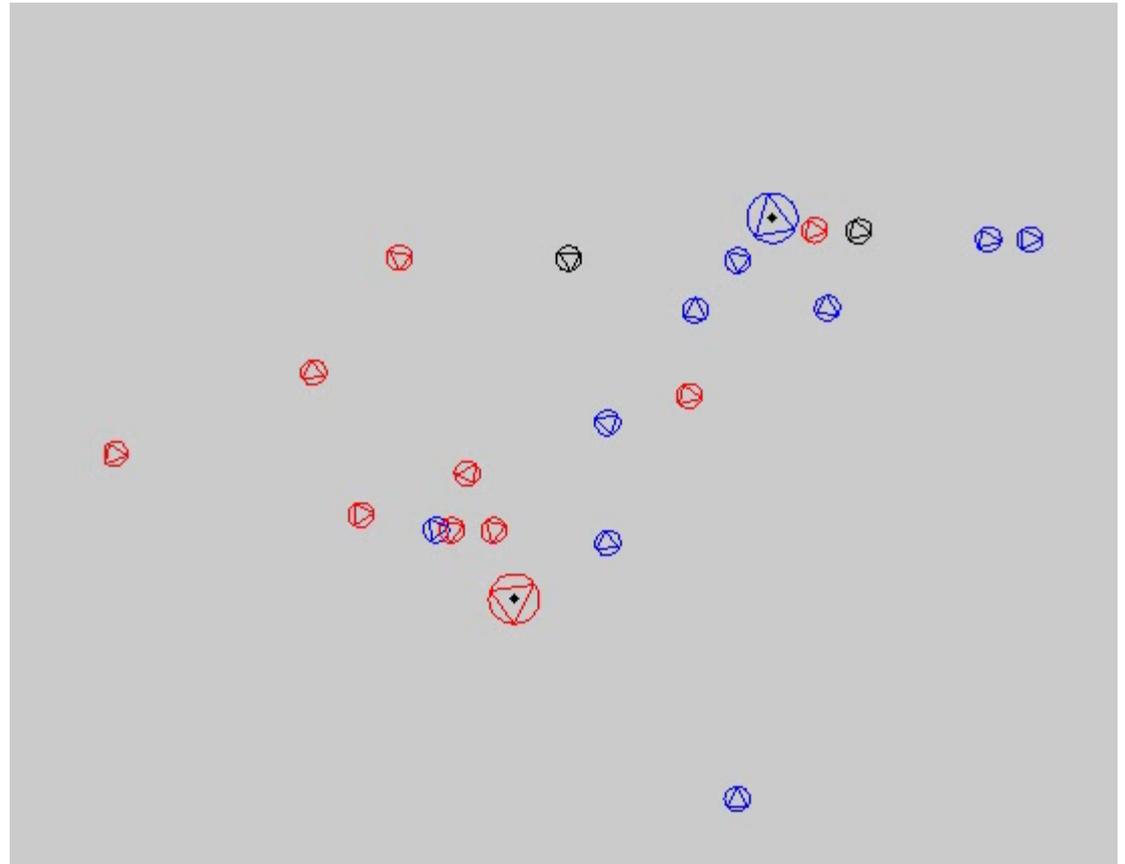
- Select command with minimum linear velocity
 - Compute the average of the commands
 - Compute a weighted average to equalize the convergence of the different subsets
-
- All of these are observed to converge, through implicit coordination.
 - No communication between the cameras/controllers is needed

Multi robot control with multiple cameras (H)



Multi robot control with multiple cameras (H)

- Multiple cameras
- Image based method which minimizes the sum of squared distances
- Partially distributed multi-robot solution for robot formation



Multi robot control with multiple cameras (H)



Hybrid multirobot control with multiple cameras, M. Aranda, Y. Mezouar, G. López-Nicolás, C. Sagüés, American Control Conference (ACC), IEEE – pages 6323-6329 (ISBN: 978-1-4799-0176-0), Washington DC, USA, 17-19 June 2013.



Reconnaissance des Formes et Perception
Intelligence Artificielle

Atelier - Perception pour le véhicule intelligent (P. Vasseur)

Visual Servoing and Exploration for Mobile Robots

Y. Mezouar

C. Sagues (G. Lopez-Nicolas, H. M. Becerra, M. Aranda)



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